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Residual Stresses in Dynamic Fatigue of Abraded Glass

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The effect of residual contact stresses on the dynamic fatigue response of glass surfaces containing abrasion flaws is described. Indentation fracture theory is used as a theoretical basis for analyzing the experimental data. The results highlight a general need to incorporate a residual-stress term in the fatigue equations for failure from natural flaws.

A RECENT dynamic fatigue study of glass disks containing well-defined indentation cracks has shown that residual contact stresses can be an important factor in strength determination.[1] As-indented disks were found to be considerably weaker than control disks subjected to a post-indentation anneal, particularly at low stress rates, indicating a strong residual-field enhancement of slow crack growth to failure. With an appropriate residual stress intensity factor incorporated explicitly into the fracture mechanics formulation, the effect could be quantified to a high degree of accuracy. The conclusions drawn from the results were shown to have important implications in the application of conventional flaw concepts to the analysis of prospective fatigue response, in particular to the use of macroscopic crack velocity data for predicting in-service lifetimes.

The ensuing question of special interest here is, how closely may the indentation crack systems described in the earlier study be taken as representative of the flaws which control the strength of brittle materials under normal conditions of preparation? Most ceramic components are surface-finished by machining, abrasion, or polishing—all contact-related processes. Accordingly, a series of dynamic fatigue tests was run on glass disks from the previous source, but with the surfaces this time containing abrasion damage rather than indentation cracks. Thus, by taking "calibrated" fracture equations from the indentation study as a baseline for comparison, it was hoped to assess the relative magnitude of the residual-stress effect for a "realistic" flaw configuration.

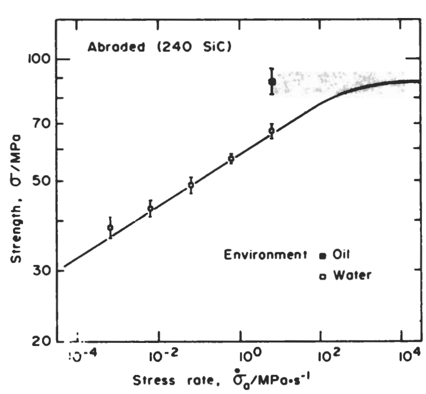

Fig. 1. Dynamic fatigue of soda-lime glass disks broken in water. *Shaded band* indicates inert strength, as determined by tests in oil; solid curve is evaluation from Eq. (2), using parameters $n=17.9$, $\nu_0=2.4\,\mathrm{mm\cdot s^{-1}}$, $X_rP/K_c=68\,\mu\mathrm{m}^{3/2}$, $K_c/(\pi\Omega)^{1/2}=0.78\ \mathrm{MPa\cdot m^{1/2}}$, and $c'_0=54\ \mu$m.

The disks (50 mm in diam. by 3 mm thick) were subjected to the following damage treatment and strength test procedure. First, a small central area ($\approx$8 mm in diam.) on each disk was grit-blasted with 240-mesh SiC particles, using an abrasion apparatus built to ASTM specifications.[2] The amount of grit used in each blast was kept sufficiently small that overlap between neighboring impact sites was negligible. Some of the abraded disks were annealed ($\approx$2 days at 520°C) to provide residual-stress-free controls. The central target areas were then covered with either distilled water, for fatigue testing, or paraffin oil, for determining "inert" strengths. The disks thus treated were immediately broken in center-symmetric flexure (outer support 39.6 mm in diam., inner support 16.0 mm in diam.), with the abraded surfaces on the tension side, at prescribed stressing rates. The results of these tests are shown as the data points in Fig. 1, each error bar representing the standard deviation for 10 to 30 disks.

Also plotted in Fig. 1 is a curve generated from the indentation fatigue equations.[1] The starting point of the analysis is an appropriate stress intensity factor for contact-induced cracks in subsequent tensile loading,

$$K=\chi_r P/c^{3/2}+\sigma_a(\pi\Omega c)^{1/2} \tag{1}$$

where c is a characteristic crack dimension, P is the peak contact load, σ_a is the tensile stress applied in the strength test, and χ_r and Ω are dimensionless geometrical parameters. This equation differs from that used in conventional fatigue analysis, in that it contains an additional, residual-stress term, characterized by the parameter χ_r. Coupled with the standard crack velocity relation $\nu=\nu_0\ (K/K_c)^n$, where K_c is the toughness and ν_0 and n are constants, along with $\sigma_a=\dot{\sigma}_a t$, where t is test duration and $\dot{\sigma}_a$ is stress rate (constant for dynamic fatigue), Eq. (1) gives

$$\dot{c}/\nu_0=\{[\chi_r P/K_c]/c^{3/2}+[(\pi\Omega)^{1/2}/K_c]\dot{\sigma}_a c^{1/2}t\}^n \tag{2}$$

For any fixed contact conditions, this differential equation must be solved numerically for the time to failure (i.e. time for K in Eq. (1) to reach K_c) at each stress rate, from which the required dynamic fatigue function $\sigma(\dot{\sigma}_a)$ may be determined. The problem thus reduces to one of expressing the given contact conditions in terms of the parameters of Eq. (2), including the initial crack dimension required for the integration procedure. In principle, these parameters may all be evaluated from control strength tests, provided the contact event is sufficiently

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well defined. However, unlike the crack patterns produced in static contact with standard hardness indenters, the abrasion damage pertinent to the present work is *not* sufficiently well defined; in particular, information on the contact load and characteristic crack dimensions for the dominant abrasion flaw is not directly available from the test observations themselves and has to be determined by calibration.

Accordingly, the parameters used in computing the theoretical curve in Fig. 1 were obtained from strength data on the abraded disks, in both the as-impacted and annealed states, along with corresponding data on annealed Vickers-indented disks from the previous study.[1] The kinetic terms $n=17.9\pm0.5$ and $v_0=2.4\pm0.6$ mm·s^{-1} follow immediately from Ref. 1, by best-fit analysis of the dynamic fatigue data for annealed indentations. The remaining terms are evaluated from inert strength behavior, the formulation for which is obtained by inserting the equilibrium condition $K=K_c$ into Eq. (1); thus

$$\sigma_a=[K_c/(\pi\Omega c)^{1/2}][1-\chi_r P/K_c c^{3/2}] \tag{3}$$

which has a stress maximum at

$$\sigma_m=3K_c/4(\pi\Omega c_m)^{1/2} \tag{4a}$$

$$c_m=(4\chi_r P/K_c)^{2/3} \tag{4b}$$

In the limit $\chi_r\rightarrow0$, $c_m\rightarrow0$, the crack reaches the point of instability without any precursor extension from its initial size c'_0† at a critical stress $\sigma^0_i=\sigma_a(c'_0)$ (defining the inert strength for zero residual stress) whence Eq. (3) reduces to

$$\sigma^0_i=K_c/(\pi\Omega c'_0)^{1/2} \tag{5}$$

From the indentation study, measurements of σ^0_i on annealed disks containing cracks of known size c'_0 gave $K_c/(\pi\Omega)^{1/2}=0.78\pm0.02$ MPa·m$^{1/2}$. For similar tests on the abraded disks in the current work, $\sigma^0_i=106\pm4$ MPa, which corresponds to an effective flaw size $c'_0=54\pm7$ μm. In the case of nonzero χ_r, the critical stress σ^r_i (inert strength for nonzero residual stress) depends on the value of c'_0 relative to c_m (Ref. 3): if $c'_0<c_m$, the flaw must undergo precursor stable growth to failure at $\sigma^r_i=\sigma_a(c_m)=\sigma_m$ ("activated" failure); if $c'_0>c_m$, the flaw propagates without limit at $\sigma^r_i=\sigma_a(c'_0)$ ("spontaneous" failure). For tests on the as-abraded disks, $\sigma^r_i=88\pm8$ MPa, so that if this quantity were to be identified with σ_m in accordance with the first of the above conditions, Eq. 4(*a*) would give $c_m=44\pm10$ μm. However, this result violates the proviso $c'_0<c_m$, so the inert strength characteristics for the abrasion flaws must be evaluated from Eq. (3); combination with Eq. (5) accordingly gives

$$\sigma^r_i=\sigma^0_i[1-(\chi_r P/K_c)/c'^{3/2}_0] \tag{6}$$

whence $X_r P/K_c=68\pm28$ μm$^{3/2}$ is evaluated.‡

The agreement obtained between experimental data and theoretical prediction in Fig. 1, notwithstanding the levels of uncertainty in the parameter determinations, supports the contention that residual contact stresses are an important consideration in general strength analysis. Comparison of the σ^0_i and σ^r_i values obtained for abraded disks above indicates a 17% reduction in inert strength due to the presence of residual stresses, and this reduction is expected to be greater for fatigue strengths.[1] Thus a (logarithmic coordinate) force fit to the data points in Fig. 1 on the basis of conventional dynamic fatigue theory (i.e. on the assumption $\chi_r=0$) yields an effective crack velocity exponent $n=15.7\pm0.7$ for the glass/water system, which is significantly lower than the true value $n=17.9\pm0.5$. For as-indented disks, on the other hand, the corresponding effective value was found to be $n=13.7\pm0.2$, so the abrasion flaw appears to possess a smaller residual-stress component than its indentation-crack counterpart. Part of the explanation of this difference could be the observed tendency for the abrasion flaws to show more extensive chipping about the impact site, thereby affording some relief of the built-in stresses (i.e. an effective reduction in χ_r). If this were the case, then surface flaws in high-density populations with a large degree of neighbor overlap (e.g. as in machining damage) might be expected to be even less affected by a residual-contact component. All surface preparations involving contact-related processes are nevertheless susceptible to the effect described here, to a greater or lesser extent, and appropriate χ_r terms accordingly need to be individually specified in any complete analysis of fatigue properties.

†Inclusion of the prime is to indicate that the crack may have undergone subcritical extension from its equilibrium size c_0 immediately after contact to c'_0 as a result of exposure to a moist environment *prior* to strength testing (Ref. 1).

‡From the previous measurements of σ^r_i on as-indented disks, knowledge of the contact load allowed the evaluation $K_c/\chi_r=31\pm4$ MPa m$^{1/2}$ to be made from Eqs. (3) and (4). This result corresponds to an effective contact load $P=2$ N for the abraded disks.

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